A SUPERCONDUCTING-DIELECTRIC RESONATOR AT W-BAND*

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ABSTRACT

This paper describes the electromagnetic properties of a superconducting-dielectric resonator (SDR) at W-band. We report that a fairly high factor (in excess of 10^5-10^6 at cryogenic temperature) for a resonator based on a sapphire tube loaded with two plates of Y-Ba-Cu oxides (its chemical composition is $Y_1Ba_2Cu_3O_7$, zero resistance at $T_c \gtrsim 80°K$). Resonators of this type have potentially valuable application such as ultrahigh stability low phase noise oscillators, descrimininators.

INTRODUCTION

During the past decades several superconducting cavities have been developed.[1]~[3] It is its very high Q that makes it very useful in many ways[4]. But to obtain the ultrahigh Q requires much efforts to prepare the cavity.

It has already been shown by Braginsky et al.[5] that the very low loss tangent of sapphire makes a dielectric resonator suitable as a high Q. For cylindrical resonators of sapphire ($\varepsilon_r \simeq 10$) enables "whispering gallory" modes to exist, with low radiation losses and consequently high Q value can be obtained if the wavelength is sufficiently small, compared with the radius of curvature. But at W-band that structure is too small to be manufactured. On the contrary, quasilight systems are overlarge if theirs' Q-values are maintained. Taking those circumstances into consideration, we would like to use a dielectric material tube guiding waves on its surfaces.

We show here a practical configuration for a SDR. at W-band. It consists of a cylindrical dielectric tube with two end plates of Y-Ba-Cu oxides mounted inside of a Cu cavity shown in Fig.1 .

For this system the effect of metallic wall on Q is very small. It can have a fairly high Q if λ_L is sufficient small (e.g. $\lambda_L \sim 10^{-5}$ cm). A detail analysis shows that Q-factor of $10^5 \sim 10^6$ may be achieved.

ANALYSIS AND CALCULATIONS

We want to find a solution satisfying Maxwell's equations (1) and the boundary conditions. We assume that each medium has the permeability of μ_0, the value for vaccum.

$$\nabla \times E = -\mu_0(\partial H/\partial t), \quad \nabla \times H = \varepsilon(\partial E/\partial t) + j \qquad (1)$$

In the superconductor the current dnesity consists of two parts according to the two-fluid model:[6]

$$j = j_n + j_s \qquad (2)$$

with $j_n = \sigma E$ for the normal part according to the classical skin effect. The supercurrent density j_s satisfies London's equation:

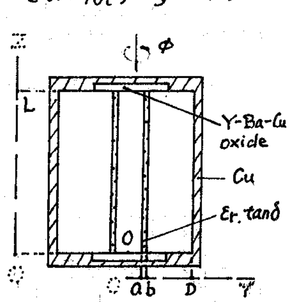

Fig.1 Cross sec. of the SDR.

$$\nabla \times j_s = -\lambda_L^{-2} H, \quad \mu_0(\partial j_s/\partial t) = \lambda_L^{-2} E \qquad (3)$$

where λ_L is the penetration depth in the superconductor. With the configuration of Fig.1. we now consider the $H_{01\delta}$ mode. Solutions can be found in the z direction of the form:

$$F(r,z,t) = F(r) \cdot \exp(i\omega t - \gamma_z z) \qquad (4)$$

where F represents field distribution function. Therefore, the determination of a solution of the partial differential equation (1)-(3) in the form eq.(4) reduces to the determination of F(r) from ordinary differential equations as follows:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dH_{zj}}{dr}\right) + K_{cj}^2 H_{zj} = 0, \quad j = 1, 2, 3 \qquad (5a)$$

$$K_{cj}^2 = \begin{cases} \omega^2\mu_0\varepsilon_j - i\omega\mu_0\sigma_j - \lambda_L^{-2} + \gamma_{zj}^2, & j = 1,3 \\ \omega^2\mu_0\varepsilon_2 + \gamma_{zj}^2 & j = 2 \end{cases} \qquad (5b)$$

We require $Re(\gamma_{z2})=0$, $Im(\gamma_{z2})=\beta_{z2}>0$. In general, $\lambda_L^{-2} \gg \omega^2\mu_0\varepsilon_0$ & $\lambda_L^{-2} \gg \omega\mu_0\sigma$, if $T < T_c$, j=1,3. Consequently, for $K_{cj} > 0$, γ_{z2} must be a positive real number. The surface resistance of the superconductor can be defined as:

$$Z^{(s)} = (E_\phi / H_r)\big|_{z=const.} \qquad (6)$$

Substituting eq.(5) into (6), we get $Z^{(s)} = \frac{1}{2}\omega^2\mu_0^2\sigma\lambda_L^3 + i\omega\mu_0\lambda_L = R^{(s)} + iX^{(s)}$. We have reasons to assume that $R^{(s)} = \frac{1}{2}\omega^2\mu_0^2\lambda_L^3$ is smaller than $(\omega\mu_0\sigma_{cu}/2)^{1/2}$, if we take λ_L as 10^{-5} cm, which is true for the metallic superconductors. At room temperature, $\sigma > \sigma_{cu}/30$, where σ and σ_{cu} are the conductivities of the oxide and the copper respectively. Making use of field matching method, we can directly obtain the following equations:[7],[8],[9]

$$\frac{C J_0'(k_{c2n}^{(2)}a) - J_0(k_{c2n}^{(2)}a)\frac{I_0'(k_{c2n}^{(1)}a)}{I_0(k_{c2n}^{(1)}a)}}{C N_0'(k_{c2n}^{(2)}a) - N_0(k_{c2n}^{(2)}a)\frac{I_0'(k_{c2n}^{(1)}a)}{I_0(k_{c2n}^{(1)}a)}} = \frac{C J_0'(k_{c2n}^{(2)}b) - J_0(k_{c2n}^{(2)}b)\frac{W_0'(b)}{W_0(b)}}{C N_0'(k_{c2n}^{(2)}b) - N_0(k_{c2n}^{(2)}b)\frac{W_0'(b)}{W_0(b)}} \quad (7)$$

$$\mathrm{delt}\left|[a_{nm}^{II}]^{-1}[b_{nm}^{II}] - [a_{nm}^{I}]^{-1}[b_{nm}^{I}]\right| = 0 \quad (8)$$

where : $W_0(b) = W_0(r)\big|_{r=b} = \left(I_0(k_{c2n}^{(1)}r)K_0(k_{c2n}^{(1)}D) - I_0(k_{c2n}^{(1)}D)\cdot K_0(k_{c2n}^{(1)}r)\right)\big|_{r=b}$

$C = k_{c2n}^{(2)}/k_{c2n}^{(1)}$, $(k_{c2n}^{(i)})^2 = \omega^2\mu_0\varepsilon^{(i)} - \beta_{zn}^2$,

$[a_{nm}^{I}] = [a_{nme}^{(1)}]^{-1}\cdot[a_{nme}^{(2)}] - [a_{nmh}^{(1)}]^{-1}[a_{nmh}^{(2)}]$,

$b_{nm}^{I} = a_{nm}^{I}\big|_{a_{nm}^{(2)} = b_{nm}^{(2)}}$; $a_{nme}^{(1)} = \exp(-3)\,\Delta_{om}/k_{c1n}$

$$a_{nme}^{(2)} = \left(\frac{\Delta_{1m}\cdot J_0(k_{c2n}^{(2)}a)}{k_{c2n}^{(1)}\cdot I_0(k_{c2n}^{(1)}a)} + \frac{\Delta_{2am}}{k_{c2n}^{(2)}} + \frac{\Delta_{3m}J_0(k_{c2n}^{(2)}b)}{k_{c2n}^{(1)}W_0(b)}\right)\cdot\sin\varphi_n$$

$b_{nme}^{(2)} = a_{nme}^{(2)}\big|_{\Delta_{2am}=\Delta_{2bm},\ J_0(k_{c2n}^{(2)}x)=N_0(k_{c2n}^{(2)}x)}$,

$a_{nmh}^{(1)} = a_{nme}^{(1)}\big|_{\exp(-3)=\alpha_{zn}\exp(-3)}$, $a_{nmh}^{(2)} = a_{nme}^{(2)}\big|_{\sin\varphi_n=\beta_{zn}\cos\varphi_n}$

$b_{nmh}^{(2)} = b_{nme}^{(2)}\big|_{\sin\varphi_n=\beta_{zn}\cos\varphi_n}$;

$\Delta_{om} = \int_a^D J_0'(k_{c1n}r)J_1(\frac{\tilde{\mu}_{1m}}{D}r)\,r\,dr = \langle J_0'(k_{c1n}r), J_1(\frac{\tilde{\mu}_{1m}}{D}r)\rangle\big|_a^D$

$\Delta_{1m} = \langle I_0'(k_{c2n}^{(1)}r), J_1(\frac{\tilde{\mu}_{1m}}{D}r)\rangle\big|_0^a$, $\Delta_{3m} = \langle W_0'(r), J_1(\frac{\tilde{\mu}_{1m}}{D}r)\rangle\big|_b^D$

$\Delta_{2am} = \langle J_0'(k_{c2n}^{(2)}r), J_1(\frac{\tilde{\mu}_{1m}}{D}r)\rangle\big|_a^b$, $\Delta_{2bm} = \Delta_{2am}\big|_{J_0'=N_0'}$;

$a_{nm}^{II} = a_{nm}^{I}\big|_{\varphi_n=\beta_{zn}L+\varphi_n}$, $b_{nm}^{II} = b_{nm}^{I}\big|_{\varphi_n=\beta_{zn}L+\varphi_n}$;

$J_1(\tilde{\mu}_{1m}) \equiv 0$ & $\tilde{\mu}_{1m} > 0$, $m = 1, 2, 3, \ldots$

$J_0(x)$, $N_0(x)$; $I_0(x)$, $K_0(x)$ are zero order Bessel's functions.
To make Q-value as high as possible requires that the electromagnetic energy should be stored in dielectric material as much as possible, i.e.

$$(p\pi - \beta_{z1}L - \Phi_1)_{\omega_0} = \min_{\omega}(p\pi - \beta_{z1}L - \Phi_1) \quad (9)$$

where p is a positive intergal (e.g. p=5). Combining (9) with (8) and (7) , we can easily find the resonant frequency and Q-value as a function of dimensions,dielectric constant and dielectric loss tangent.[10] The calculated values are plotted in Fig. (2), (3) and (4).

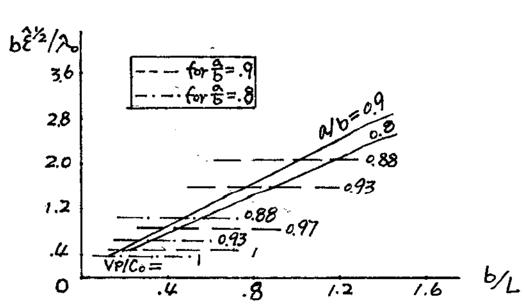

Fig.2 Resonant frequency as a function of geometries and $\hat{\varepsilon}_r = \varepsilon_r/2.08$

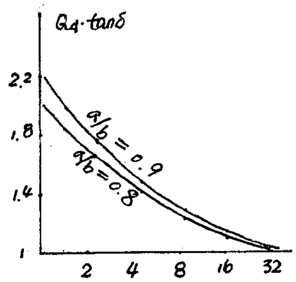

Fig.3 Q_d as a function of $\tan\delta$ and ε_r

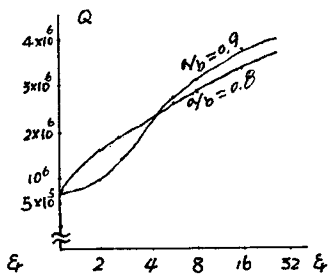

Fig.4 Residual losses as a function of λ_L and ε_r

A room temperature counterpart is designed with a teflon tube ($\tan\delta = 5.8\times10^{-4}$, $\varepsilon_r = 2.08$), 7.92 mm long, 5.72 mm and 6.36 mm in inner and outer diameters. The H_{015} mode is excited at 94.9 GHz in the resonator. The calculated Q is about 4277. While for the SDR., the dielectric tube is made of a sapphire one (at T=80°K, $\tan\delta \lesssim 10^{-5}$, and $\lambda_L = 10^{-5}$ cm, $\varepsilon_r = 9.88$), with 2.475 mm and 2.75 mm in inner and outer diameter respectively its Q-value is in the range of 10^5-10^6; and it is much higher than that of the room temperature resonator. Preliminary experimental results of the room temperature resonator shows that it is feasible for millimeter waves applications.

CONCLUSION

The SDR described here is physically small, compact, rigid, light weight, easily in manufacture and fairly high quality factor if a low loss tangent dielectirc material is used (e.g. sapphire).

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